IS THERE SOMETHING TO EXPECT FROM GAME THEORY?*

For somebody studying economics, game theory is quite mysterious: almost everywhere, “allusions” are made to it (for instance, in organisation theory, in labour theory, in international trade economics, in public economics, etc.), but one can obtain a B.A. (and, even, a master) degree in economics without having followed a special course in this theory – so important and so full of promises, it is said. One can find in recent microeconomic textbooks – Varian and Schotter, for example – one or two chapters on game theory, but they limit themselves to some little stories (almost always the same: prisoners’ dilemma, battle of sexes, entry deterrence, store chain paradox, centipede game) and a rough definition of Nash equilibrium. Indeed, quite surprisingly, these examples seldom lead to clear conclusions or “results”; on the contrary, they either present dilemmas or paradoxes, or they imply that Nash equilibria are solutions to certain problems.

As they are full of mathematical symbols, game theory books, or textboks (as Osborne and Rubinstein’s, or Fudenberg and Tirole), are very difficult to understand. All game theory presentations, advanced or not, have however a common feature: they *never* give concrete examples, with real data (coming from observed situations and facts). In the place of data, figure other, more or less complicated stories, that are summarised either by a table (the “strategic form” of the game), or by a “tree”, with branches and nodes (its “extensive form”). These stories (almost) always center on the same question: is there a rational way to make a decision when gains depend on this decision, but also of others’ decisions? Indeed, there are a lot of situations where there is no simple answer – or no answer at all – to this question. This is why there are so many “dilemmas” or “paradoxes” among game theory stories. Game theory doesn’t “resolve” problems, concrete or not: it highlights the difficulty to caracterize rational behavior. Ariel Rubinstein, a recognized game theorist, is totally right when he explains that “game theory is a fascinating and abstract discussion that is closer to philosophy than to the economic pages of the newpapers. It is for this reason that there are no undergraduate game theory courses: if students in

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economics are told, from the beginning, that there is often no clear answer to the question: “what is a rational decision?”, even in quite simple - but interdependent - situations, they will then be very skeptic when their professors (specially in microeconomy) tell them that they are going to study “rational choices” and their consequences. Think of an economic cursus with students critizising each "solution" proposed because it does'nt "really" result from rational behavior …

Games as stories

Game theory has been invented by mathematicians – it is often presented as a branch of mathematics. This is its principal source of prestige: isn’t mathematics synonymous of riguour, and their results (theorems) indisputable? But it is also its main weakness: game theory models are always “stories”, like fables or parables, with no relation with real life situations. When mathematics are used, all aspects of models must be translated in symbols (sets, functions, equations, etc.). But, as no situation in social life can be reduced in such way, game theorists (as microeconomists) invent stories, called *games* – because they are like parlour games, where people are isolated from real world, know almost everything about each other and about possible outcomes, and only have to respect imposed rules. If it is so, there can be precise calculations – such as determining the players’ payoffs.

Game theory models are thus *all* of the same type; that is, they are *all* formed by three ingredients:

1. *Individuals*, called *players*, who want to maximize their payoffs (they are *rational*).
2. *Sets*, whose elements are called *strategies.* Each player has one set of strategies, and player i is supposed to choose one element s in his set of strategies, S_i ($i = 1, ... , n$, if there are n players).
3. *Rules*, that associate an *outcome* of the game with each *strategy profile* ($s_1, ... , s_i, ... , s_n$), $s_i \in S_i$, $i = 1, ..., n$, and that assign payoffs to each outcome. Rules are often represented by *payoff functions*. If $g_i(\cdot)$ is the payoff function for player i, $g_i(s_1, ... , s_i, ... , s_n)$ gives his payoff when the profile of strategies is ($s_1, ... , s_i, ... , s_n$).

Which strategies will players then choose? Before answering this question, it is necessary to precise what each player knows about others, and about the rules of the game (and thus, about payoffs in each outcome).

Game theorists assume, at least for a start, that there is *complete information*: everybody knows everything about everybody, including that they are rational (all this being *common knowledge*)[1]. It is very clear that *there is no real life situation which fulfills these conditions* (1, 2 and 3, plus complete information); it is why game theory models are only stories, without real data – as parlour games.

A fundamental point

All game theory models are formed by ingredients 1, 2 and 3 (differences in models come from differences in sets of strategies, in payoff functions and, sometimes, in players' information). When these conditions are fulfilled, players are supposed to choose *separately and simultaneously* one element of their strategy set, and then *the game is over*: the outcome corresponding to these choices (and, thus, players' payoffs) is determined. Assuming that choices are unique and made simultaneously is *fundamental* in game theory; without this assumption, outcomes will not be well defined. But, again, it is quite difficult – or impossible – to find real life situations where such an assumption can be accepted. And, moreover, usual, or "popular", presentations of game theory completely ignore it, especially when they explain that, thanks to game theory, it is possible to analyse "strategic behavior", "conflict" or "cooperation", with "threats" and "retaliations", and so on. How can this be possible when players decide – separately and simultaneously – once, and only once?

In fact, confusion proceeds here from two sources: first, game theorists have an interest to "sell" their models, by insinuating that they can explain many aspects of real life (like "strategic interactions", "conflicts", etc.); the other source of confusion comes from the fact that it is possible to construct games fulfilling conditions 1, 2 and 3 with more than one move (sequential games and repeated games), but with simultaneous and unique choices. In this kind of game, strategies are "lists of instructions", about actions choosen by players at each move (at each node of the game tree). Strategies are, thus, "conditional", as they are of the kind: "if I am in this situation, then I will do that; if I am in this other situation, then I will do that; and so on".

[1] They also consider "incomplete information" games, which are not fundamentally different, but much more complicated (it is supposed that there are players (or payoffs) "types", with some probabilities - both being common knowledge), especially because some kind of beliefs are included in models' parameters, or " fundamentals ".

Each player then chooses one list of instructions (a strategy) in his strategy set (whose elements are lists of instructions), all players doing the same, at the same time. When all lists of instructions are choosen, and known by everybody, players – or some kind of referee – determine what happens at each move ("a path in the game tree") and, thus, every player's payoff.
For instance, consider the prisonners' dilemma game, played twice. Each player's strategy set has then four elements, like: "I won't implicate my colleague at the first move; at the second move, I will implicate him if he has implicated me at the first move but I won't implicate him if he has not implicated me at the first move", or: "I implicate my colleague at the first move ; at the second move, I implicate him again, whatever has he done at the first move".
Intuition is here misleading: it seems natural to consider that, like in real life, players decide successively, at each move, after observing what has happened in the precedent moves (and then "learning" from that). But game models don't proceed in this way, and we know why: players are supposed to choose simultaneously one, and only one, strategy (of the conditional type when there is more than one move). *This is not intuitive at all* and, again, there is no real life situation of this type; for instance, it is impossible to find an example of an oligopoly, an entry with deterrence, or any kind of competition with threats, cooperation, etc. reduced to unique and simultaneous choices – even if textbooks, or books alluding to game theory "analysis" and "results", create confusion when they speak of "dynamics" about game models with many moves. Indeed, these books never present clearly what we have called the "fundamental point" (i.e. unique and simultaneous choices). If they did so, a normal person would stop reading, as there is no interest (except for fun) to continue with so counterintuitive, and misleading, stories.

About games "solutions"

Game theory is a collection of stories that need to satisfy conditions 1, 2 and 3, and thus irrelevant for real life situations. But it is always possible to *create* situations such as those described in their stories, and then ask: which strategy will rational players choose in this or that situation?
If there is a simple answer to this question, then it can be considered as "the solution" of the game, the outcome predicted by the theory. This can happen if there is an oucome where players' payoffs are bigger than in all other oucomes. But this is a very specific case. Moreover, it is not

interesting, as game theory is concerned with conflicts, where players disagree about the game's "best outcome" (as they get their bigger payoff in different outcomes).

Prisonner's dilemma is another case, or story, whose solution seems to be obvious ... when, in fact, it isn't (it is a "dilemma")! The story is described in figure 1, the first number in couples (*a,b*) gives player *A*'s payoff, the second number *B*'s payoff.

		B	
		don't implicate	*implicate*
A	*don't implicate*	(1, 1)	(–2, 2)
	implicate	(2, –2)	(–1, –1)

Figure 1

There seems to be an obvious "solution" for this game, because the strategy "implicate your colleague" dominates the strategy "don't implicate him" – you get greater payoffs if you "implicate" than if you "don't implicate", when the other implicates you ($-1 > -2$) or when he doesn't ($2 > 1$). If players *A* and *B* are rational, they must then choose both to "implicate" their colleague ... but then their payoffs are lesser than if they have both choosed to stay mute. "Rational" choice doesn't lead to a "rational" outcome.

The dilemma is quite more dramatic when the prisonners' dilemma is repeated *n* times (*n* being "as big as one wants", but finite). The new game is a game with *n* moves (each move being described by figure 1) – strategies being then lists of instructions of the conditional type "implicate (or don't) if ..." Here too, there is an "obvious" solution: "implicate your colleague until the last move, whatever has he done before", as this strategy dominates "iteratively" the others. Proof is given by *backward induction*: the last move situation is the same than in the "one move" prisonners' dilemma: whatever has happened in the past, "implicate" dominates "don't implicate". In the $n-1$ move, both players give the instruction: "implicate", as it dominates "don't", and as they know that what they do then has no incidence on last move decision. The same happens in the $n-2$ move, and so on, until the first move.

Such a solution, apparently so logic, is in fact absurd: prisoners stay all the time in jail (payoff : $-n$) when they could be free (payoff : n)! Is this solution a "prediction" of the theory – in a positive mood? Or is it the choice proposed by the theorist, in a normative mood? The answer is, of course, "no" in both cases. Can theorist predict any other outcome that could then be considered as a "solution" of the game? No, as choices depend of expectations: if a player doesn't implicate his colleague in certain moves (certain nodes of the game tree), it is because he expects that he will not be implicated by him. His list of instructions thus depends on his expectations about the list of instructions choosen by his colleague; and as these expectations can be of any form (they depend, among other things, on the opinion that both have of each other), precise predictions are not possible. The same can be said from the normative point of vue.

Games solutions and rationality

Prisonners' dilemma played only one time has a predictable outcome, because players have a dominant strategy. They both know that, and each expect that, the other will choose his dominant strategy. Consequently, they are even more convinced that they have to choose their own dominant strategy. It is easy to expect what the other's decision will be. But games where every player have a dominant strategy are of a very particular kind. In general, games have no dominant strategies, as in the game described in figure 2.

			A	
		a_1	a_2	a_3
	b_1	(2,2)	(1,6)	(4,1)
B	b_2	(3,3)	(1,2)	(1,0)
	b_3	(2,1)	(7,6)	(3,7)

Figure 2

Is there a solution for this (very simple) game, if by "solution" we mean a specific outcome (a strategy profile) predicted by theory? No, because all outcomes can be the result of rational players' choices – game theorists say that all players' strategies are *rationalizable*. For instance, *A* is rational in choosing strategy a_1 if he expects that *B* will choose b_3; and *B* is rational in choosing b_3 if he expects that *A* will choose a_3; this last choice is rational if *A* expects that *B* will choose b_2 which is rational if *B* expects that *A* will choose a_1, and so on. Indeed, in this game, all strategies are "rationalizable": there are reasons to choose each of them (actual choices depend of beliefs about others' choices). If "rationalizability" is the only condition required to be a solution of a game, then all outcomes of the game in figure 2 are solutions of this game and no precise prediction is possible (the only prediction is: anything can happen).

From the two very simple examples (or stories) described in figures 1 and 2, we can deduce two importants lessons.

1. In general, assuming that players are rational doesn't imply a unique prediction.
2. When there is only one outcome "rationalizable", it may not be "collectively rational", and then an unfullfilled prediction (like in repeated prisoner' dilemma).

Normally, we should stop here : as game theory is interested in determining rational people's "decisions in interaction", and as, in general, assuming a player's rationality is insufficent to make predictions different than : "anything, or almost, can happen".

But game theorists don't do that: they want to publish papers, to get a job as profesors and, probably, they like to invent stories and discuss about their "solutions" or the "dilemmas" that they can raise. Special attention is then given to particular outcomes, *Nash equilibria*, where expectations are as important as rationality.

Nash equilibrium as a result of correct expectations

In the figure 2 example, there is a couple of strategies, $\{a_2, b_1\}$, different from other outcomes, in the sense that each players have "correct" expectations: A chooses a_2 because he thinks that B will choose b_1, and B chooses b_1 because he thinks that A will choose a_2. A couple (or a profile) of strategies where players anticipate correctly what others will do is, by definition, a *Nash equilibrium*. As in a Nash equilibrium each player maximises his payoff, given others' choices, Nash equilibria satisfy in a certain way a rationality condition. But, as in prisonner' dilemma (simple or repeated), "individual rationality" doesn't imply "social rationality": in our example, players' payoffs are bigger with $\{a_3, b_2\}$ than with $\{a_2, b_1\}$. It may then happen that Nash equilibria are not "best outcomes", thus they cannot be considered as "solutions" from a normative point of vue.

But also, often, they cannot be justified from a positive point of vue – that is, Nash equilibria cannot be considered as predictions of what (rational) players will do. This is obvious in repeated prisoner's dilemma (where there is only one Nash equilibrium: each player always implicates his colleague), especially if it is repeated many times (say, more than ten times). In the figure 2 example, it is possible to consider that $\{a_3, b_2\}$ is a prediction at least as good as $\{a_2, b_1\}$ (Nash equilibrium): a_3 is *A'*s choice because he thinks that *B* is going to choose b_2 where payoffs are, "in the mean", higher than those can be obtained with b_1 and b_3; there is a risk, for *A*, that *B* will choose b_3, but even in this case his payoff is not less inferior than that of equilibrium (indeed, $\{a_3, b_3\}$ is not a bad prediction, too).

Cournot duopoly model, one of game theorists' favorite models, gives another example where Nash equilibrium is a very bad prediction. In this model, each duopolist (player) only knows his own *reaction function*, which gives his product supply in response to the other duopolist's supply. As always in game theory, choices (here, supplies) are made *simultaneously* (like in all game models). What can be predicted about duopolists' choices? Nothing: as they don't know anything about other's choice, the only thing that they can do is to choose randomly their supplies – i.e., anything can happen.

Why Nash equilibrium?

Game theorists, and people who speak about game theory, focus their attention on Nash equilibrium. They try to see if it exists (in pure or mixed strategies), if it is unique, if it is "robust" (not too sensitive to parameter's value), etc. But, they seldom answer the question: why give so much importance[2] to Nash equilibrium? Indeed, the use of the word "equilibrium" is quite misleading as, in general, the idea of equilibrium is associated with that of process – equilibrium being the "result", the "outcome" of a process (its "resting point"). But this is nonsense in game theory, because there are no processes at all, players' choices being – by assumption – unique (and simultaneous).

Cournot's model gives an example of how, often, game theorists and, always, textbooks justify in a wrong way Nash equilibrium, as a "solution" of a game. They sketch *both* reaction curves in the *same* figure, and then focus attention on the point where they intersect. Isn't this point the « obvious » solution of the model? Indeed, the only way to present the model in a correct manner is to draw one figure with one duopolist's reaction curve (this is what this duopolist knows) and

[2] The same can be said about equilibrium in Stackelberg model (where one of the player, is "cournotian" and then chooses his supply randomly). In the other duopole model, Bertrand's, rational players will never choose Nash equilibrium strategies, where payoffs are equal to zero: if they propose a price bigger than equilibrium price, they can have a strictly positive payoff (it depends of the price proposed by the other duopolist).

another figure with the other duopolist's reaction curve. Each duopolist then chooses one point in his reaction curve, and there is no reason that these points are "precisely" the points where the two curves intersect (indeed, the probability that this happens is zero). It is then clear that Nash (or Cournot) equilibrium *is not* a prediction of the model, and that there is *no* reason to give it such importance. Often, textbooks present Cournot equilibrium as the result of a process: one duopolist makes an offer, at random; the other "reacts" to this offer, and makes his own offer; in his turn, the first duopolist "reacts" to this offer, and so on, until they reach the equilibrium (the point where reactions curve intersect). But this is nonsense, because rational duopolists will modify their reaction curves as the process goes on (as they notice that their competitor reacts to their offers), and thus the equilibrium (intersection point), changes during the process, as each of them observes how the other reacts (or plays) – equilibrium is "path dependent", and thus indeterminate.

In summary: in most of their "stories" (and, especially, in those of the "imperfect competition" type, as duopole models), game theorists are unable to answer the question: why do you pay so much attention to Nash equilibrium? They cannot justify, then, all the mathematics that they use in discussing the "proprieties" of these specific outcomes (the only justification is that they are interesting, from a mathematical point of vue).

Game theory and "experiments"

Games, in game theory, are simple stories: they don't describe (even approximately) observed situations. But it is always possible to make "experiments", asking people to "play the game", as in parlour games. There a lot of "experiments" of this kind. What are their conclusions? That "real" people, in flesh and blood, often don't react as the theory predicts (when it predicts something). Even in the case of the simple prisonner's dilemma, not repeated, there is a minority of people who choose the strategy "don't" implicate, and it seems that they are not rational. When the game is repeated only two or three times, few people choose what seems to be the "rational" strategy ("always implicate"). But the most famous example is the so-called "ultimatum game": player *A* says to player *B*: "Somebody will give me a cake only if you accept to share it with me. Now, I propose to give you *x*% of the cake. Do you accept?"

If *B* is rational, he must accept even if *x* is tiny (it is better to have something than nothing). If *A* is rational, and thinks that *B* is rational too, he will then propose an *x* near zero: in this case, there is a quite clear prediction. But “experiments” don’t confirm it: in general, people like *B* don’t accept propositions if *x* is far from 50%, and people like *A* don’t propose a tiny *x*. Game theorists explain this by sense of equity: if people feel that the share is “too unfair”, they prefer to diminish their gains than to accept. Everybody agrees with them, but we can deduce that if this is true in a such simple situation, it must be true in a lot of real life situations, where people live together, interact – and where payoffs are not only monetary. Indeed, allmost all “experiments” in game theory don’t confirm the predictions of the theory, even when these predictions are well defined: people don’t act as they are supposed to do.

Recent game theory litterature is full of another kind of “experiments” in the so-called, and fashionable, “evolutionary game theory”. The starting point of this theory, is that people *don’t choose* : each player is identified with a strategy (often, a conditional one). We are then at the opposite side of the starting point of game theory, which is to try to determine how rational people *choose* (or can choose) one of their (many) strategies.

It’s amazing to see that game theorists – so proud of their “rigor” – use the same words (game theory) to design completely different theories (even if they have some formal similarities). Indeed, in “evolutionary game theory” each individual is reduced to a strategy, and their are “tournaments” where strategies are confronted, two by two. Payoffs give the number of “offsprings”, who are at their turn confronted in “tournaments”, and so on, until some kind of “equilibrium”, where some (or all) strategies “survive”, is reached.

Countless “experiments” of this type can be made: only a computer and some imagination are needed! Strategies can be very sophisticated (including some kind of “learning”, decisions in each move depending on what happened before that move) or very simple. Theory doesn’t predict anything; theorists only choose the strategies and rules of the “tournament” that they will play (thanks to computers), and then comment the “results” obtained – whatever they are. Mathematicians can try to find which strategies are “evolutionary stable” in this or that “tournament”, and so on. But it remains to prove that these new kind of “stories” are of any interest – in biology as in social science.

Microeconomics as a branch of game theory

Microeconomy textbooks are, like those in game theory, full of little stories with fictitious households and fictitious firms,which interact in fictitious markets – and with no data (when there are some, they are irrelevant, because they are of the aggregate kind). Indeed, microeconomy must be considered as a branch of game theory as its' purpose is to study interactions of rational agents' decisions. Game theorists are, however, more rigorous than current microeconomists, as they give more importance to the rules of the game than to the players' "tastes" or "psychology". When microeconomists speak of "price mecanisms", "market forces", "flexibility", etc., game theorist ask : "what do yo mean by that ? Please, say exactly what is the strategic set of each player, what kind of information do they have, how do they interact, etc.".

Take microeconomist's preferred model: perfect competition. One of it most important assumption is that households and firms are "price takers"; their strategic sets are then bundles of goods. A game theorist asks: "who sets prices?", and microeconomists have to recognize that it is (implicitly) supposed that there is "somebody" setting prices, and that these prices are known, and accepted, by households and firms. The game theorist would pursue by saying: "OK, but now you have to precise this 'person' payoffs". Microeconomist (for example, Arrow and Debreu) says : "Well, it is a benevolent person whose only 'satisfaction' is to make lower as possible other's agents total excess demand, in value". The game is then well defined: households and firms announce bundles of goods, anticipating their prices; the price setter anounce a price vector, anticipating (total) good's excess demand, each one wanting to get a maximum payoff. If, after (simultaneous) announcements, agents notice that they have correctly anticipated other's choices, then there is equilibrium. You can call "competitive" this (Nash) equilibrium – even if it is not clear at all where is "competition" in those simultaneous, and unique, announcements –, but it is sure that it supposes a quite curious institutional arrangement ("game's rules") and, incidentaly, its probability as an outcome of players' choices is zero (how can anyone predict exactly other's choices, knowing nothing about them ?).

Conclusion

When somedy speaks of game theory and its "results", or "insights", first ask: "could you please give me a real life situation with observed facts and data that can be discribed as a game, in game theory's sense?". If the answer that you are given is of the kind: theory always simplifies, it tries to explain "stylised facts", to understand what rational choices can be in different kinds of situations (catching some important aspects of real life), then ask : "OK, but then, can you tell me what are the predictions of game theory, its proposed 'solutions'?". If your interlocutor replies: "well, the first thing to do is to see if there is (at least) a Nash equilibrium", then insist : "Do you mean that Nash equilibrium is the prediction of the model, the result of players' rational choices, its 'solution'?", and wait for the answer ... If it doesn't come, or if it is confused, then close your eyes and your ears, and refuse all the figures and maths studying "properties" of Nash equilibria, and so on.

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